Control Charts For Variables

BEM3104 Engineering Quality Control

Engineering Management Program
Faculty of Engineering and Industrial Technology, SSRU

Variation

- There is no two natural items in any category are the same.
- Variation may be quite large or very small.
- If variation very small, it may appear that items are identical, but precision instruments will show differences.

3 Categories of variation

- ***WITHIN-PIECE VARIATION***
  - variation found within a single item or unit measured at different points or portions.
- ***PIECE-TO-PIECE VARIATION***
  - differences observed between individual units or items that are supposed to be identical.
- ***TIME-TO-TIME VARIATION***
  - the changes in measurements or outcomes that can occur over time due to temporal fluctuations or external factors impacting the process.

Source of variation

- **EQUIPMENT**
  - *Tool wear, machine vibration, …*
- **MATERIAL**
  - *Raw material quality*
- **ENVIRONMENT**
  - *Temperature, pressure, humadity*
- **OPERATOR**
  - *Operator performs- physical & emotional*

Control Chart Viewpoint

- Variation due to
  - Common or chance causes
  - Assignable causes
- Control chart may be used to discover "assignable causes"

Some Terms

- **RUN CHART** - without any upper/lower limits
- **SPECIFICATION/TOLERANCE LIMITS** - not statistical
- **CONTROL LIMITS** - statistical

Control Chart Functions

- Control charts are powerful aids to understanding the performance of a process over time.

What’s causing variability?

Control Charts Identify Variation

- Chance causes - "common cause"
  - inherent to the process or random and not controllable
  - if only common cause present, the process is considered stable or "in control"
- Assignable causes - "special cause"
  - variation due to outside influences
  - if present, the process is "out of control"

Control Charts Help Us Learn More About Processes

- Separate common and special causes of variation
- Determine whether a process is in a state of statistical control or out-of-control
- Estimate the process parameters (mean, variation) and assess the performance of a process or its capability

Control Charts to Monitor Processes

- To monitor output, we use **a** control chart
  - we check things like the mean, range, standard deviation
- To monitor a process, we typically use **two** control charts
  - mean (or some other central tendency measure)
  - variation (typically using range or standard deviation)

Sources of Variation

- Many factors contribute to variation
- **Source of variation - combination of equipments, materials, environment, operator, etc.**
- **Equipment -** tool wear, electrical fluctuations for welding
- **Material -** tensile strength, moisture content (e.g. raw material)
- **Environment -** temperature, light, humidity etc.
- **Operator -** method, SOP followed, motivation level, training
- **Inspection -** inspector, inspection equipment, environment

Causes Of Variation - Chance & Assignable

- Chance or random causes are unavoidable
- As long as fluctuate in natural/expected/stable pattern of chance causes of variation which are small – it is OK
- This is in ‘state of statistical control’
- When causes of variation large in magnitude; can be identified, classified as assignable causes of variation. If present, process variation is excessive (beyond expected natural variation)
- ‘state of out of control’ – assignable cause
- Example : Body temperature - 36.5^{o}C ~ 37.5^{o}C

Control Chart Method

- Control chart - means of visualizing variations that occur in the central tendency and dispersion of a set of observations
- Graphical record of a particular quality characteristic – hardness, length, etc

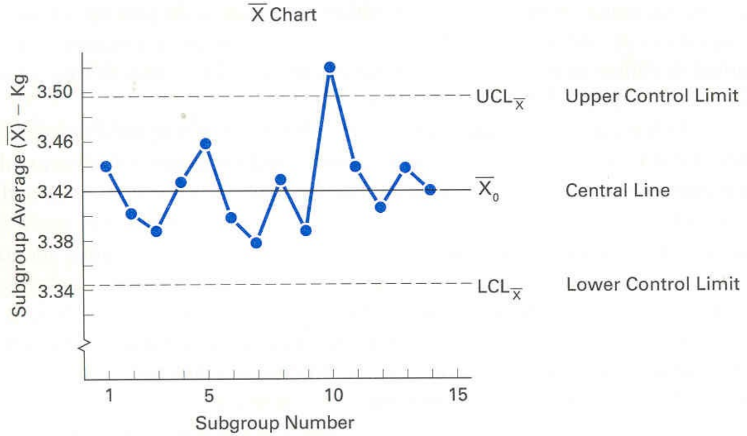

FIGURE 5-1 Example of a control chart.

Control Chart Method

- Control limits are not specification limits
- CL are permissible limits of a quality characteristic
- Evaluate variations in quality subgroup to subgroup
- Limits established at ± 3 standard dev. from central line; for normal distribution – we expect 99.73% of items would lie within the limits

Control Chart Method

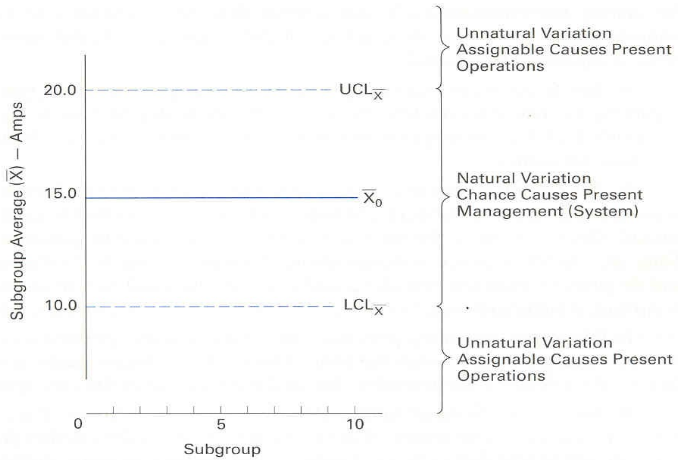

FIGURE 5-3 Natural and unnatural causes of variation.

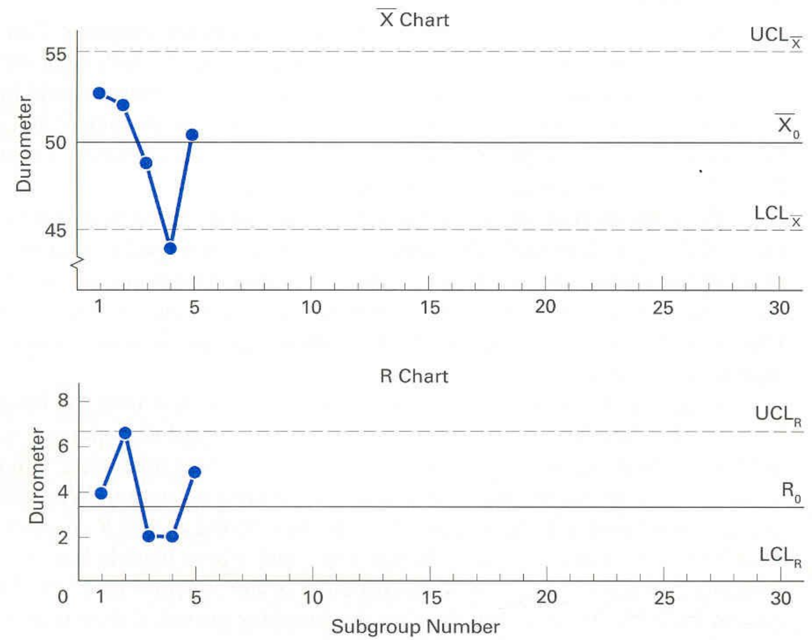

FIGURE 5-2 Example of a method of reporting inspection results.

Types of Data

- **VARIABLE DATA**
  - Product characteristic that can be measured
    - Length, size, weight, height, time, velocity

- **ATTRIBUTE DATA**
  - Product characteristic evaluated with a discrete choice
    - Good/bad, yes/no

Control Chart for Variables

- Variables are the measurable characteristics of a product or service.
- Measurement data is taken and arrayed on charts.

Control Charts for Variables

- **X-bar chart**
  - In this chart the sample *means* are plotted in order to control the mean value of a variable (e.g., size of piston rings, strength of materials, etc.).
- **R chart**
  - In this chart, the sample *ranges* are plotted in order to control the variability of a variable.
- **S chart**
  - In this chart, the sample *standard deviations* are plotted in order to control the variability of a variable.
- **S^2 chart**
  - In this chart, the sample *variances* are plotted in order to control the variability of a variable.

X-bar and R charts

- **THE X- BAR CHART** is developed from the average of each subgroup data.
  - – used to detect changes in the mean between subgroups.
- **THE R- CHART** is developed from the ranges of each subgroup data
  - – used to detect changes in variation within subgroups

Control chart components

- **CENTERLINE**
  – shows where the process average is centered or the central tendency of the data
- **UPPER CONTROL LIMIT (UCL) AND LOWER CONTROL LIMIT (LCL)**
  – describes the process spread

Variable Control Chart – $\bar{x}$ (average)- R chart

1. Select quality characteristic

- Measurable data (basic units, length, mass, time, etc.)
- Affecting performance, function of product
- From Pareto analysis – highest % rejects, high production costs
- Impossible to control all characteristics - selective or use attributes chart

2. Choose rational subgroup

- Rational subgroup - variation within the group due only to chance causes and can detect between groups changes
- Two ways selecting subgroup samples
  1. Select subgroup samples at one instant of time or as close as possible
  2. Select period of time products are produced

- Rational subgroup from homogeneous lot : same machine, same operator
- Decisions on size of sample empirical judgment + relates to costs
- • choose n = 4 or 5 $\rightarrow$ use R-chart
- • when $n \geq 10$ $\rightarrow$ use s-chart
- frequency of taking subgroups often enough to detect process changes
- Guideline of sample sizes/frequency using
- • Say, 4000 parts/day

$\therefore$ 75 samples

- • if n = 4 $\therefore$ 19 subgroups
- • or n = 5 15 subgroups

TABLE 5-1 Sample Sizes (From ANSI/ASQ Z1.9—1993, Normal Inspection, Level II)

LOT SIZE	SAMPLE SIZE
91–150	10
151–280	15
281–400	20
401–500	25
501–1,200	35
1,201–3,200	50
3,201–10,000	75
10,001–35,000	100
35,001–150,000	150

3. Collect data

- Use form or standard check sheet
- Collect a minimum of 25 subgroups
- Data can be vertically / horizontally arranged

	Subgroup Number								
Measure	1	2	3	4	5	……	…..	….	25
x_1 x_2 x_3 x_4 x_5	35 40 32 37 34								34 40 38 35 38
$\bar{x}$	35.6								37.0
R	8								6

EXAMPLE PROBLEM

TABLE 5-2 Data on the Depth of the Keyway (millimeters)[a]

SUBGROUP NUMBER	DATE	TIME	MEASUREMENTS X_1	X_2	X_3	X_4	AVERAGE $\bar{X}$	RANGE R	COMMENT
1	12/26	8:50	35	40	32	37	6.36	0.08	
2		11:30	46	37	36	41	6.40	0.10	
3		1:45	34	40	34	36	6.36	0.06	
4		3:45	69	64	68	59	6.65	0.10	New, temporary
5		4:20	38	34	44	40	6.39	0.10	operator
6	12/27	8:35	42	41	43	34	6.40	0.09	
7		9:00	44	41	41	46	6.43	0.05	
8		9:40	33	41	38	36	6.37	0.08	
9		1:30	48	44	47	45	6.46	0.04	
10		2:50	47	43	36	42	6.42	0.11	
11	12/28	8:30	38	41	39	38	6.39	0.03	
12		1:35	37	37	41	37	6.38	0.04	
13		2:25	40	38	47	35	6.40	0.12	
14		2:35	38	39	45	42	6.41	0.07	
15		3:55	50	42	43	45	6.45	0.08	
16	12/29	8:25	33	35	29	39	6.34	0.10	
17		9:25	41	40	29	34	6.36	0.12	
18		11:00	38	44	28	58	6.42	0.30	Damaged oil line
19		2.35	35	41	37	38	6.38	0.06	
20		3:15	56	55	45	48	6.51	0.11	Bad material
21	12/30	9:35	38	40	45	37	6.40	0.08	
22		10:20	39	42	35	40	6.39	0.07	
23		11:35	42	39	39	36	6.39	0.06	
24		2:00	43	36	35	38	6.38	0.08	
25		4:25	39	38	43	44	6.41	0.06	
Sum							160.25	2.19	

[a] For simplicity in recording, the individual measurements are coded from 6.00 mm.

4. Determine trial control limits

- Calculate Central line
- $\bar{X} = \frac{\sum_{i=1}^{g} \bar{x}_i}{g}$ $\quad R = \frac{\sum_{i=1}^{g} R_i}{g}$
- $\bar{X}$ = avg. of subgroup avg.
- $\bar{x}_i$ = avg. of ith subgroup
- g = no. of subgroups
- $\bar{R}$ = avg. of subgroup ranges
- R_i = range of ith subgroup
- Where A2, D4, D3 are factors
  - vary according to different n

$$UCL_{\bar{x}} = \bar{\bar{x}} + 3\sigma_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R}$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - 3\sigma_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R}$$

$$UCL_R = \bar{R} + 3\sigma_{\bar{x}} = D_4\bar{R}$$

$$LCL_R = \bar{R} - 3\sigma_{\bar{x}} = D_3\bar{R}$$

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5. Revised Control Limits

- **First plot preliminary data collected using control limits & center lines established in step 4**
- **Use/adopt standard values, if good control i.e. no out-of-control points**

$$\bar{\bar{X}} \rightarrow \bar{X}_o \qquad \bar{R} \rightarrow R_o$$

- **If there are points out-of-control discard from data, look at records – if show an assignable cause – don't use**

$$\bar{\bar{X}}_{new} = \frac{\Sigma\bar{x} - \Sigma\bar{x}_d}{g - gd}$$

$$\bar{R}_{new} = \frac{\Sigma R - \Sigma Rd}{g - gd}$$

$$\sigma_o = \frac{R_o}{d_2}$$

Control Charts with Limits Established

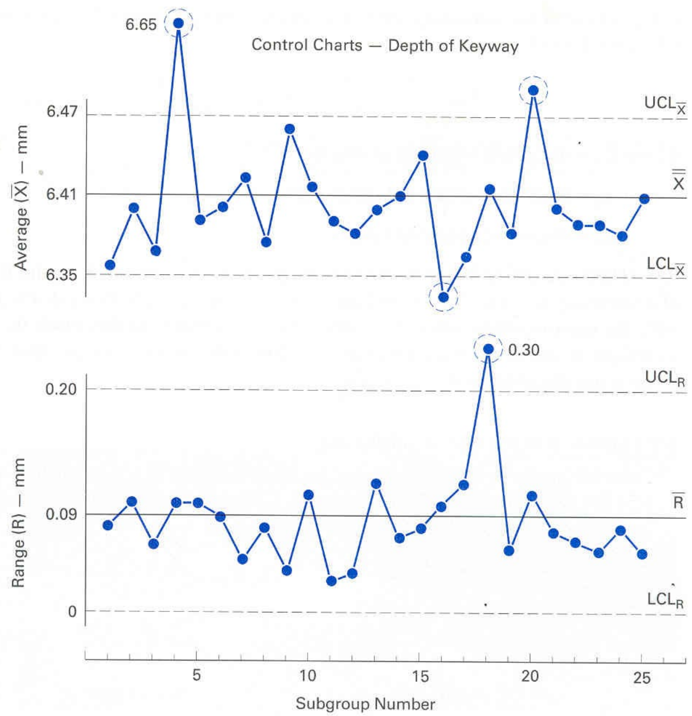

FIGURE 5-5 $\bar{X}$ and R chart for preliminary data with trial control limits.

- Limits for both charts become narrower after revised
- Revised limits used to report / plot future sub-groups
- For effective use – chart must be displayed and easily seen

Comments about CC

1. Some analyst eliminate the revised step - but actually more representative of process
2. Formula mathematically same $\overline{X}_o + A\sigma_o = \overline{\overline{X}}_{new} + A_2\overline{R}_{new}$
3. Initial estimate of process capability known - $6\sigma o$ - true Cp is next
4. If use specification; nominal (target) value = $\overline{X}o$. Range doesn't change
5. Adjustments made to processes while taking data – not necessarily continue making defectives while collecting data
6. Process determines center line and the control limits, not design or manufacturing
7. When population values known easily obtained limits,

$$\overline{X}_o = \mu \cdot \sigma_o = \sigma$$

6. Achieving objective

- Initiate control charts results in quality improvement
- Less variation in sub-group averages
- Reduction in variation of range
- Can reduce frequency of inspection - monitoring purpose – even once/mth.

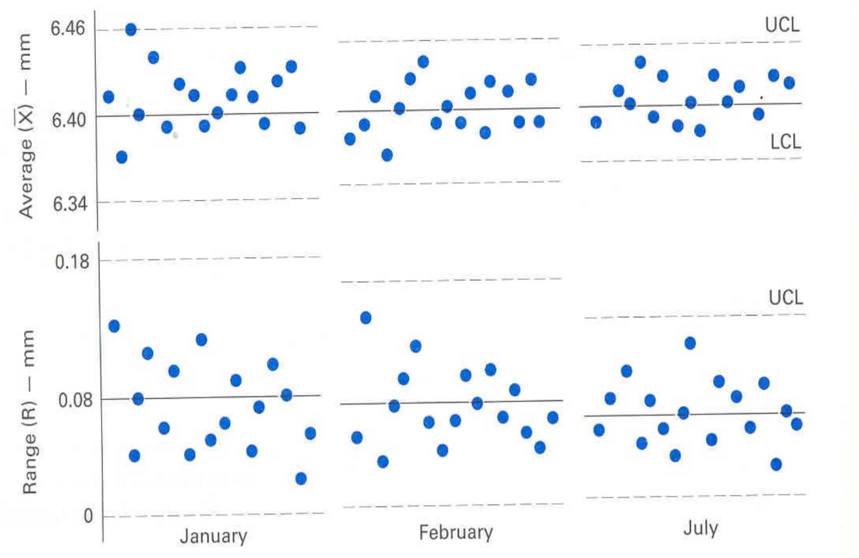

FIGURE 5-7 Continuing use of control charts, showing improved quality.

How Control Chart Helps in QI

- Psychological effect to do better – example - maintenance group helps adjust process center
- Purchasing involved in changing material supplier to ensure consistent quality
- Production – standardize work methods, use/develop new tooling
- Improvements must be from investigation of assignable causes (need technical back up)

Sample Std. Deviation Chart
($\bar{x}$ - s control chart)

- Both R and s measure dispersion of data
- R chart - simple, only use X_H (highest) and X_L(lowest)
- s chart - more calculation
  - use ALL xi's
  $\therefore$ more accurate, need calculate sub-group sample standard deviation
- When n < 10

  R chart $\cong$ s chart
- n $\geq$ 10 - s chart better , R not

accurate any more

$$UCL_{\bar{x}} = \bar{\bar{X}} + A_3\bar{s}$$

$$LCL_{\bar{x}} = \bar{\bar{X}} + A_3\bar{s}$$

$$UCL_s = B_4\bar{s}$$

$$LCL_s = B_3\bar{s}$$

$$\sigma_o = \frac{s_o}{C_4} = \frac{\bar{s}}{C_4}$$

State of Control

- When assignable causes eliminated and points plotted are within C.L.- process state of control
- Further improvement through changing basic process, system
- What are the characteristics of process in control? (natural pattern of variation)
- 34% within 1σ from Center Line
- 13% between 1σ & 2σ
- 2.5% of plotted points - 2$\sigma \rightarrow$ 3σ
- Points located back & forth across center line random way
- No points out of control

- Subgroup averages forms frequency distribution which is normal distribution and limits – established at 3σ from center line.
- Choice of 3σ is economic decision with respect to 2 types of error
- Type I - occurs when looking for assignable cause but in reality chance cause present > FALSE ALARM
- When limits set $\pm 3\sigma$ Type I error probability = 0.27% or 3/1000
- Say point our control $\rightarrow$ due to assignable but 3/1000 of the time can be due to chance cause
- Type II - assume chance cause present, but in fact assignable cause present > TRUE ALARM
- Records indicate 3σ limits balance between 2 errors.

Process in Control

- Individual parts will be more uniform – less variation and fewer rejects
- Cost of inspection will decrease
- Process capability easily attained
- Trouble can be anticipated before it occurs
- Percentage of parts fall between two values can be predicted with highest degree of accuracy, e.g. filling machines
- $\bar{X}$-R charts can be used as statistical evidence for process control

- Predictable and stable process only chance causes present

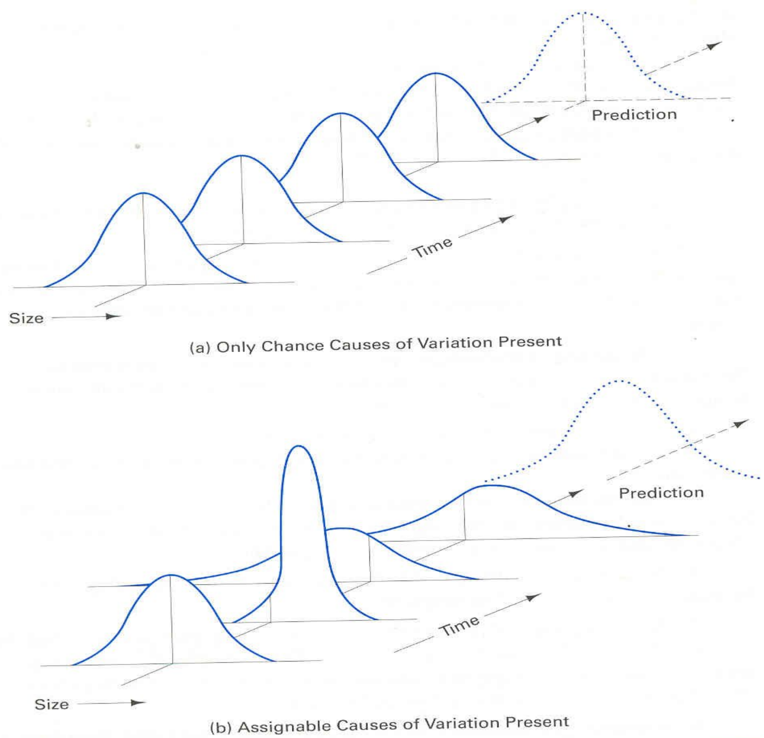

FIGURE 5-10 **Stable and unstable variation.**